Argumentation and Reasoned Action
Proceedings of the 1st European Conference on Argumentation,
Lisbon 2015

Volume II

Edited by
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Reasoning Types and Diagramming Method

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By associating and combining Ajdukiewicz’s classification of reasoning with argumentation diagrams and schemes we show how to represent a rich variety of reasoning types with different degrees of complexity as inference, derivation, justification, verification (i.e. confirmation and falsification) or explanation. We also indicate some meta-schemes concerning the process of reasoning itself, and we discuss diagrams and meta-schemes assigned to abduction.

KEYWORDS: abduction, argumentation diagrams, argumentation schemes, derivation, explanation, inference, justification, metalanguage, reasoning, verification

1. INTRODUCTION

One of the original achievements of the Lvov-Warsaw School in methodology, which can be an object of interest of the argumentation theory, is the development of the classification of reasoning (cf. Koszowy & Araszkiewicz, 2014). It was primarily introduced by Łukasiewicz (1912), who followed Twardowski’s ideas. Then it was improved by Czeżowski (1946). Finally, the classification of reasoning was essentially modified and elaborated by Ajdukiewicz in (1955), which seems to be the most advanced approach. The publications and the ongoing discussion were carried out in Polish, but the main ideas and their development are exhaustively surveyed by Kwiatkowski (1993) in English.

The aim of this paper is to use Ajdukiewicz’s classification in the argumentation theory in order to associate reasoning types with argumentation schemes and diagrams, which are usual tools for representing and analyzing natural language arguments in informal logic (cf. Reed, Walton, & Macagno, 2007; Walton, Reed, & Macagno, 2008).
First we recall the definitions of reasoning and some basic reasoning types as they were distinguished by Ajdukiewicz, namely the definitions of inference, derivation, justification, verification and explanation. In the next part of the paper we discuss the relationship between argumentation and reasoning, and in order to distinguish representations of the reasoning types considered by Ajdukiewicz we develop the notation of argument structure offered by ‘the standard diagramming method’ (cf. Jacquette, 2011). By determining epistemic and heuristic status of sentences we point out diagrams assigned to various reasoning types. We also show that some of these diagrams need to have schematic components. Finally, we indicate some meta-schemes concerning the process of reasoning itself, and we discuss diagrams assigned to abduction.

2. AJDUKIEWICZ’S CLASSIFICATION OF REASONING

Łukasiewicz and Czeżowski distinguished exactly four reasoning types, which have the same level of complexity: inference, justification, confirmation (positive verification) and explanation. All of them are simple in the sense that they do not consist of any proper part which is a reasoning too.

Ajdukiewicz (1955) presented a criticism of Łukasiewicz and Czeżowski’s approach and developed his own classification, which is essentially different than the criticized one. In Ajdukiewicz’s classification inference and derivation are basic reasoning types, which are components of all other ones. Thus, in order to grasp the whole variety of reasoning types we must begin with the definitions of inference and derivation.

2.1 Definition of reasoning

Ajdukiewicz defined inference as a kind of thinking (i.e. mental) act in which acceptance is transferred between sentences.

To infer means to come, on the basis of some accepted sentences (propositions), to the acceptance of a new, not yet accepted sentence (proposition), i.e. conclusion, or to increase, on a basis of some accepted sentences, the certainty, with which another sentence is accepted (Ajdukiewicz, 1955, p. 282).

In contrast to inference from actually accepted premises, derivation is a suppositional form of reasoning, which involves
sentences merely assumed, that is potentially, hypothetically accepted, i.e. suppositions. Ajdukiewicz called it expressively a “pseudo-inference”. Thus, “to derive” means to come, on the basis of some potentially accepted sentences, to the potential acceptance of a new sentence.

Inference and derivation can be purely spontaneous, but they can be also goal-driven. Namely, they can be controlled by a kind of thinking task in this way that they are components of reasoning, which is used to complete such a task. Thus, Ajdukiewicz’s definition of reasoning takes the following, non-classical, disjunctive form:

we propose to classify as reasoning: 1) any processes of inference; 2) processes of derivation, that is of “pseudo”-inference, 3) processes of solving thinking tasks and problems, carried out by using inference or derivation (Ajdukiewicz, 1955, p. 294).

Reasoning is a thinking, i.e. mental act, which can be wrong, incorrect, erroneous, faulty or mistaken in any other sense, thus it does not need to be deductive or satisfy any conditions of logical or extra-logical correctness at all. Empirical course of reasoning processes, however, is the matter of psychological research. Since in this paper we consider the logical structure of reasoning, we do not focus on thinking acts themselves, but on their representations in a language, namely on sentences and their possible inferential orderings.

2.2 Thinking tasks and goal-driven reasoning

Thinking tasks can be expressed by a question or by an imperative sentence. First let us take into account justification, i.e.

a thinking process of solving a task, which requests a sentence totally given in this task to be inferred from other already accepted sentences (Ajdukiewicz, 1955, p. 282).

The task of justification is given by the imperative ’prove (demonstrate) Q’. In order to complete this task, a sentence (or sentences) P is to be found such that Q can be inferred from P, and moreover the process of inferring Q from P must be realized.

A type of reasoning controlled by a question is verification. It consists in
the acceptance of a sentence [...] or of its negation [...] based on determining the truth or falsity of some consequences derived from this sentence (Ajdukiewicz, 1955, p. 282).

The question for verification has the form "is $P$ true?". The given sentence $P$ is not yet accepted, so that it cannot be a premise of any inference. Therefore, it is used as a supposition, and a sentence $Q$ is derived from it, such that $Q$ happens to be an either accepted or refused sentence (we assume that sentences are refused iff their negations are accepted). Finally, if $Q$ is accepted then $P$ is inferred, and if $Q$ is refused then $\neg P$ is inferred (and thereby $P$ is refused). Thus, verification can take any of two forms. It can be either confirmation or falsification, according to whether the sentence given in the task is eventually accepted or refused.

The task for explanation is given by the question "why $P$ (why is $P$ true)?". The thinking process leading to answer this question can be divided into three stages: 1) finding out such a sentence (or sentences) $Q$, that another given and already accepted sentence $P$ can be derived from $Q$, and next, after deriving it, 2) inferring the obtained new sentence $Q$ from the given sentence $P$, unless $Q$ happens to be already accepted, and finally 3) inferring the answer of the form "$P$ because $Q$" (cf. Ajdukiewicz, 1955, p. 283). Since the second step is not always necessary, we can distinguish two types of explanation. If, say, a theft explains to someone his car disappearance, then the sentence about the theft was not yet accepted by him, and it must be inferred. This is an example of the first type. But if the explanation is that his wife took the car, and he can even recall when she did it, then the explanatory sentence is already accepted, and it need not to be inferred. This is an example of the second type of explanation. The case when a well-known, accepted theory serves us to explain some phenomenon belongs to the second type as well.

3. DIAGRAMMATIC REPRESENTATION OF REASONING

3.1 Reasoning and argumentation. Classical diagramming method

In opposition to reasoning, which is a thinking act (or process), we understand argumentation as an act (or process) of communication, i.e. a speech act, in which some agent’s thinking acts of reasoning are presumed and submitted to an audience for acceptance. The tasks assigned to reasoning types are explicitly present in argumentation dialogue, and they are also expressed in suitable speech acts. Following Hitchcock (2007) who claims that Pinto’s “happy” phrase “Arguments
are invitations to inference (Pinto, 2001)" applies also to suppositional reasoning (i.e. derivation), we are inclined to extend this suggestive characteristics and understand arguments as invitations to reasoning, which can take any of three forms specified by Ajdukiewicz's definition.¹

If we disregard the persuasive and performative function of argumentation, we obtain propositional structures, which are the same in arguments and in the presumed acts of reasoning. Therefore the schemes and diagrams assigned to reasoning types and to the corresponding arguments are the same too. We will show how to adapt the classical diagramming method to represent the whole variety of reasoning types, which is potentially included in the definition of reasoning given by Ajdukiewicz. Trzęsicki (2011) proposed an alternative diagramming notation of reasoning, however, it corresponds to the classification formulated by Czeżowski. Firstly because derivation is absent in Trzęsicki’s notation. Furthermore, in accordance with his proposal, inference is represented by one of four basic types of diagrams. Justification, explanation and confirmation (but not falsification, which is neglected) correspond to the remaining three. Thus inference in Trzęsicki’s notation, as well as in Czeżowski’s (and in Łukasiewicz’s) classification, is one of four simple reasoning types, instead of being the component of the remaining three as it appears in Ajdukiewicz’s approach.

We refer to the classical diagramming method as it was described by Jacquette (2011). Possible complexity of the classical diagrams is illustrated by Figure 1.

Figure 1 – Classical argumentation diagrams

¹ Separate derivations do not form arguments in a narrower sense of the term “argument”. Since they do not have any explicitly accepted conclusion, in this sense they can be at most a kind of components of complete, proper arguments.
Complex argumentation structures, as the one presented in Figure 1e, can be obtained using the elementary operations corresponding to the constructions shown in Figures 1a-d.

3.2 Epistemic status of sentences. Inference and derivation

In order to adapt classical diagrams to Ajdukiewicz’s classification first of all we must be able to distinguish the representations of inferences from the representations of derivations. Since sentences involved in these two fundamental reasoning types have different epistemic status, all the sentences on diagrams must be indicated in some way as being either actually accepted as in inference, or merely potentially accepted as in derivation. Freeman (1991) used a dashed line circles to represent suppositions. Here, in order to indicate the epistemic status of sentences, suppositions will be enclosed in square brackets (angle brackets, although not consequently, have been introduced by Jacquette (2011) for the aim of representing ad absurdum arguments).

Simple inference and derivation are represented by Figures 2a and 2b respectively. More complex examples of derivation are shown in Figures 2c-e. The dashed arrow in 2c means that there is a series of derivations between $P_2$ and $P_n$.

Suppositions and actually accepted sentences can be combined (Figures 2d-k), however not every combination can be regarded as meaningful. Contrary to the example shown in Figure 2j, if a premise of some reasoning is a supposition, so should be the conclusion (in Jacquette’s diagrams only first premises of derivations are in brackets, what is the reason for the claim that he did not develop his notation consequently). Also convergent combinations of inference and derivation (Figure 2i) seem to be meaningless, since the epistemic status of their conclusion is unclear. However, the following reasoning shows that perhaps it is a more problematic case: God probably exists, because people believe in
God, but if he would give us some sign now, his existence would be more certain. On the other hand, one can say that there are two separate arguments in this example, and the latter is actually a meta-argument about the first one. We will not discuss this issue here. Let us only note that a possible evaluation of such reasoning seems to require a kind of hybrid semantics, in which the acceptance of each sentence is a pair of values representing its actual and its potential component. Convergent derivations (Figure 2k) do not cause similar problems.

Since derivation itself only leads to suppositions, in order to produce an actually accepted conclusion, it must be involved in some more complex reasoning process as ad absurdum or practical reasoning for instance. Furthermore, derivation is often summarized by means of natural deduction. Thus, a sequence of derivations can be concluded by an implication of a special form. This implication has the conjunction of all the assumed suppositions as its predecessor, and the sentence eventually derived as its antecessor (cf. Freeman, 1991, p. 214). Derivations represented by Figures 2c–e have been completed by means of natural deduction and shown in Figures 2f–h, respectively. The horizontal line at the arrow preceding the final implication indicates that not merely the last supposition, but the whole underlined derivation plays the role of a premise in the final inference.

It should be emphasized that by using the term "natural deduction" we do not mean that all the compound, individual derivations must be deductive. They can be defeasible as well. Here, the word "deduction" is used by analogy to the deduction theorem\(^2\), well-known from handbooks of formal logic. It is a meta-theoretic theorem which characterizes the relationship between formal derivation and implication. Thus, by "natural deduction" we mean an analogue of the deduction theorem for natural language. Obviously, the implication obtained as the conclusion of some derivation can be a premise of some further inference. It can be a premise of ad absurdum or practical reasoning, or of any other suitable reasoning as confirmation (see Sect. 3.4). So to speak, natural deduction is a "de-assumizer" – an uniform method to change the epistemic status of that what follows from suppositions.

\(^2\) If \( A \cup \{\alpha\} \trans S \beta \) then \( A \trans S \alpha \rightarrow \beta \), where \( \alpha \) and \( \beta \) are any sentences and \( A \) is any set of sentences formulated in the language of a fixed system \( S \) (\( \trans S \) is the consequence relation of \( S \)). Obviously, it depends on the particular properties of a formal system if the deduction theorem holds for it.
3.3 Heuristic status of sentences. Justification and spontaneous inference

Standard diagrams do not allow us to distinguish inference from derivation. They do not allow us to distinguish spontaneous inference from justification as well. The cause is that they do not represent the heuristic status of the sentences involved in reasoning processes, namely they do not indicate which of these sentences are given at the starting point, and which occur at the subsequent stages of reasoning. In case of spontaneous inference the given sentences are premises, but in justification they are conclusions. In order to indicate this difference we use asterisks, so that the asterisked sentences are the starting points of reasoning. Thus Figure 3a provides an example of a simple spontaneous inference, while Figure 3b represents a simple justification. Figures 3c and 3d show a complex spontaneous inference and a complex justification, respectively.

Next two diagrams in Figure 3 represent some combinations of spontaneous inference and goal-driven justification. The course of reasoning can be read from Figure 3e as follows: $P_1$ is given, $P_4$ is inferred from it, then $P_6$ is additionally justified by $P_5$ and $P_3$, then a new premise $P_s$ is found, and $P_7$ is inferred from it and from $P_6$, then $P_7$ is additionally justified by $P_6$, and eventually the final conclusion $C$ is inferred from $P_7$. In Figure 3f the sentence $P_7$ is justified firstly, and then the conclusion $C$ is inferred from $P_7$. Figure 3g is the representation of some reasoning driven by a (e.g. mathematical) task "prove $C$ from $P_1$, $P_2$ and $P_4$!".

3.4 Verification and practical reasoning

Since a supposition is the starting point of verification, diagrams assigned to this reasoning have to use both brackets and asterisks. Figures 4a and 4b show the diagramming of confirmation (without and with regard of natural deduction step, respectively). The remaining four diagrams in Figure 4 represent falsification, which can take any of two
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forms: *ad absurdum* as in Figures 4c and 4d, or *ad falsum* as in Figures 4e and 4f. The explicit use of natural deduction in Figures 4b, 4d and 4f shows clearly that the last step of verification is an inference. This inference is deductive in falsification, while in confirmation it is not, however, since in real-life reasoning the derivative part of verification (represented by the dashed arrows) is mostly defeasible, neither confirmation, nor falsification can be absolutely conclusive in this case.

![Diagram of Verification](image)

Figure 4 – Verification

Yet, let us consider representations of practical reasoning, which are shown by Figure 5. Practical reasoning is similar to verification, but the last inference has a different scheme. Also the suppositions that occur must have some special form, since they refer to some action and its possible effect.

![Diagram of Practical reasoning](image)

Figure 5 – Practical reasoning

Practical reasoning begins with the asterisked supposition assuming that we take up some action. Then we derive what will be an effect of this action. If this effect is desired, we conclude that the action should be performed (Figure 5a). If it is undesired, we conclude that the action should be prevented (Figure 5c). In Figures 5b and 5d natural deduction is used to explicitly reveal the scheme of the last, crucial inference. At the same time, the derivative part is still represented on the diagrams. Thus, by combining diagrams and schemes, the whole course of
practical reasoning can be represented more accurately than by using schemes only.

3.5 Explanation and abduction

Diagrams of explanation are similar to those of confirmation, but they involve a special scheme of conclusion. Also the starting points of both reasoning types are different. While confirmation begins with a spontaneous derivation of consequences from a given sentence, the starting point of explanation is the sentence, which is to be supposedly justified. This type of explanation is shown in Figure 6a. The second type, in which the explanatory sentence is already accepted, is shown in Figure 6b.

The final conclusion in both cases is the sentence "P₂ because P₁," which means that P₁ is true, P₂ is true, and that P₂ follows from P₁. Therefore the sentence P₁ should be regarded as an implicit premise of the final inference. This premise has been added in Figure 6c specifying the second type of explanation (Figure 6d shows a version of this type, in which the explanatory sentence is used as a supposition in the initial derivation, but then it is not being inferred, because its veracity has been determined in another way, e.g. it was recalled). A detailed diagram for the first type of explanation is more complex. Actually, we have two steps of reasoning here. At the beginning we conclude with P₁ by showing that some given, acceptable sentence P₂ is derivable from it, and next on the same basis enriched with just accepted sentence P₁ (and with derivation replaced by inference), we draw the final conclusion 'P₂ because P₁'. This structure is shown in Figure 6e.

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Figure 6 – Explanation
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The diagrams in Figure 6 can be further specified. Namely, the derivations and inferences leading from P₁ to P₂ can be concluded explicitly by the sentence "P₂ follows from P₁" (the implication "P₁ → P₂", offered by natural deduction, is too weak to entail the conclusion "P₂
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because $P_1$). Let us note, however, that "$P_2$ follows from $P_1$" belongs to semantics, so (unlike "$P_1 \rightarrow P_2$") it is a metalanguage sentence.

Since one and the same fact can have many different explanations, the inference of the explanatory sentence $P_1$ (Figure 6e) is rather unreliable, so that the whole reasoning can be regarded at most as an ad hoc explanation. Obviously, this unreliable inference can be supported by some other reasoning as e.g. falsification (Figure 7a). But it can be also enhanced by adding a premise which states that this particular explanation is the most successful one. Reasoning enriched in this way is called "abduction". In argumentation schemes of abduction this evaluative premise usually refers explicitly to some specified alternative explanations (cf. Walton, Reed & Macagno, 2008, p. 329). Figure 7b shows the diagram based on the backward scheme of abduction referring to $n$ different explanations of the same fact.

Walton, Reed, & Macagno also consider the forward scheme of abduction. Its diagrammatical representation is shown in Figure 7c. This reasoning aims at selecting the conclusion of “the most plausible (strongest) argument” having premises which belong to some given basis of plausible statements (cf. Walton, Reed, & Macagno, 2008, pp. 329-330). The term “argument” is interpreted in Figure 7c as denoting derivation, because the conclusions $Q_1, Q_2, \ldots, Q_n$ can (or even should) be mutually exclusive, so that they must not be accepted at the same time, which would be the case if they were inferred from the basis $P$. So only one of them can be selected as the conclusion of the final inference, and thereby accepted. Let us note that these sentences often refer to some mutually exclusive possibilities, which are known in advance. The task for such reasoning might be formulated as the imperative "choose (i.e.

Figure 7 – Abduction
select the most plausible statement) among \( Q_1, Q_2, \ldots, Q_n \) with respect to the basis \( P^! \). The representation of this reasoning is the diagram shown in Figure 7c, but with \( Q_1, Q_2, \ldots, Q_n \) asterisked above the horizontal line (if \( Q_i \) was asterisked below the horizontal line too, the diagram would denote a sort of justification).

The question marks in Figures 7b and 7c mean that here we intend to discuss neither the issue of "the most successful explanation", nor that of "the strongest argument (derivation)". But it has to be underlined that evaluative premises referring to some reasoning cannot be avoided in abduction, what makes it being a meta-reasoning in fact. Thus meta-schematic components are indispensable in diagrams of abduction.

Finally, let us note that arguments based on meta-schemes, i.e. on the schemes referring to some reasoning, are frequently encountered in everyday argumentation. In addition to abduction and explanation, which need components of this kind, we have various meta-schemes referring to the fact that some act of reasoning was (or was not) performed, or that it was performed in some sense correctly (or incorrectly). The conclusion in these meta-schemes can be identical with the conclusion of the reasoning under consideration (or with its negation), but it can also contain an assessment of this statement (in particular, various forms of argument from ignorance fit this description) or of the person who reasons.

4. CONCLUSION

Classical diagramming method offers a certain concept of argument structure. Introducing indicators of epistemic as well as heuristic status of sentences allowed us to expand this concept in order to grasp more of important features of everyday argumentation. In one of my previous papers I proposed a formal definition of argument, which corresponds to the classical diagrams (cf. Definition 3: Selinger, 2014, p. 382). Thus, a question arises about an extension of this definition, which would enable us to cover the entire realm of arguments revealed by Ajdukiewicz’s analysis of reasoning.

Obviously, a formal representation of epistemic and heuristic status of sentences has to be introduced in some way (e.g. by assigning pairs of Boolean values to the sentences). However, the crucial question is that of the character of argument premises. In the arguments representable by the classical diagrams only sentences can be premises. But this seems to be undermined due to the occurrence of arguments containing derivations as their counterparts. Since suppositions can lead only to suppositions, it is not the last derivative (e.g. \( P_2 \) in Figure
4a) that is the premise of the final inference. Hitchcock (2007) claims that also reasoning can be a premise of argument, and the whole derivation should be regarded as just such a premise in this case. This seems to be the most natural solution, however, it leads to a meaningful increase of the formal complexity of the definition in question. For instance, derivations may have premises which are again derivations, so that such a definition, unlike the one previously proposed in (Selinger, 2014), needs to be recursive in order to cover all possible levels of derivation. Thus, there is a question whether it is in some way possible to maintain the concept of argument, which employs only sentences operating as premises.

Another question addressed mostly to the future work is to estimate the extent to which metalanguage has to be introduced to the formal concept of argument structure. We have shown that meta-theoretic and meta-schematic elements are not dispensable in the diagrams of abduction. Introducing meta-theoretic elements to the diagrams of explanation is desirable, although not necessary. These are, however, only some partial results, therefore this issue needs a more elaborate, systematic and in-depth study.

ACKNOWLEDGEMENTS: I gratefully acknowledge the support of the Polish National Science Centre under grant 2011/03/B/HS1/04559.

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